Ind.-Anfang

Ind.-Schluss:

Anr.: Aussage richtig für 2^k

Zeige: Gilt auch für 2^{k+1}

Horner-Schema

$f(x) = 2{,}1x^4 + 3{,}$

$f(5) = 2{,}1\cdot 5\cdot 5\cdot$

4 Mult

Horner:

$f(x) = -0{,}9 + x\cdot($

$= -0{,}9 + x\cdot($

$= -0{,}9 + x\cdot($

Horner-Schema

$$f(x) = 2{,}1x^4 + 3{,}9x^3 - 0{,}2x^2 + x - 0{,}9$$

$$f(5) = \underbrace{2{,}1\cdot5\cdot5\cdot5\cdot5}_{4\text{ Mult.}} + \underbrace{3{,}9\cdot5\cdot5\cdot5}_{3\text{ Mult.}} - \underbrace{0{,}2\cdot5\cdot5}_{2} + \underbrace{1\cdot5}_{1} - 0{,}9$$

$$= 4+3+2+1 = \frac{4\cdot 5}{2}$$

Horner:

$$f(x) = -0{,}9 + x\cdot\left(1 + \left(-0{,}2x + 3{,}9x^2 + 2{,}1x^3\right)\right)$$

$$= -0{,}9 + x\cdot\left(1 + x\cdot\left(-0{,}2 + 3{,}9x + 2{,}1x^2\right)\right)$$

$$= -0{,}9 + x\cdot\left(1 + x\cdot\left(-0{,}2 + x\cdot\left(3{,}9 + 2{,}1\cdot x\right)\right)\right)$$

$f(x) = \sin x$

$$(a_k) = \left(0, 1, 0, -\frac{1}{6}, 0, \frac{1}{120}, 0, -\frac{1}{5040}, \dots\right)$$

Taylor-Reihe mit Entwicklungsstelle 0

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} x^k$$

$$= f(0) + f'(0)\cdot x + \frac{f''(0)}{2}\cdot x^2 + \frac{f'''(0)}{6} x^3 + \dots$$

Bsp $\sin(x)$

$f(x) = \sin x$

$f'(x) = \cos x$

$f''(x) = -\sin x$

$f'''(x) = -\cos x$

$f^{(4)}(x) = \sin x$

$f(x) = \sin x$

$f'(x) = \cos x$

$f''(x) = -\sin x$

$f'''(x) = -\cos x$

$f^{(4)}(x) = \sin x$

$f(0) = \sin 0 = 0$

$f'(0) = \cos 0 = 1$

$f''(0) = -\sin 0 = 0$

$f'''(0) = -\cos 0 = -1$

$$(a_k) = \left(0, 1, 0, -\frac{1}{6}, 0, \frac{1}{120}, 0, -\frac{1}{5040}, \dots\right)$$

^{3}

$x + \dots,$

$$\sin x \approx x - \frac{1}{6}x^3$$

$$\sin x \approx x - \frac{1}{6}x^3 + \frac{1}{120}x^5 - \frac{1}{5040}x^7$$