Horner-Schema:

Taylor-Reihe

$$f(x) = \sum_{k=0}^{\infty} \frac{f^{(k)}(0)}{k!} x^k$$

$$= f(0) + f'(0)\cdot x + \frac{f''(0)}{2} x^2 + \frac{f'''(0)}{6} x^3 + \frac{f''''(0)}{24} x^4 + \dots$$

$f^{(k)}$ = k-te Ableitung

$f^{(0)} = f$

$f^{(1)} = f'$

$f^{(2)} = f''$

Bsp.: Sinus

$f(x) = \sin x$ $\quad f'(x) = \cos x$ $\quad f''(x) = -\sin x$ $\quad f'''(x) = -\cos x$

$f(0) = 0$ $\quad f'(0) = 1$ $\quad f''(0) = 0$ $\quad f'''(0) = -1$

$$\sin x = \sum_{k=0}^{\infty} a_k x^k \quad \text{mit}$$

$$(a_k) = \left(0, 1, 0, -\frac{1}{6}, 0, \frac{1}{120}, 0, -\frac{1}{5040}, 0, \dots\right)$$

Approximation: $x \approx 0 \Rightarrow f(x) \approx \sum_{k=0}^{m} \frac{f^{(k)}(0)}{k!} x^k$

$$\sin x \approx x - \frac{1}{6} x^3$$

$$\sin x \approx x - \frac{1}{6} x^3 + \frac{1}{120} x^5$$

$$\sin x \approx x - \frac{1}{6} x^3 + \frac{1}{120} x^5 - \frac{1}{5040} x^7$$

Algo DS 2|2
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$$p(x) = 3{,}1\,x^4 + 5x^3 - \frac{1}{4}x^2 + x - 9$$

$$p(4) = 3{,}1 \cdot \underbrace{4 \cdot 4 \cdot 4 \cdot 4}_{4 \text{ Fakt.}} + 5 \cdot \underbrace{4 \cdot 4 \cdot 4}_{3x} - \frac{1}{4} \cdot \underbrace{4 \cdot 4}_{2} + 4 - 9$$

$$p(x) = (((3{,}1x + 5)x + (-\tfrac{1}{4}))x + 1) \cdot x - 9 \quad \leftarrow \quad \Theta(n) \text{ Mult.}$$

$3{,}1x^4 \rightarrow 4$ Mult.
$5x^3 \rightarrow 3$ Mult.
$-\frac{1}{4}x^2 \rightarrow 2$ "
$+x \rightarrow 1$ "

$$\sum_{i=1}^{4} i = \frac{4 \cdot (4+1)}{2} = 10 = \underline{\underline{\Theta(n^2)}} \text{ Mult.}$$